digital Control lec 7

م. شهاب

state space

1) Representation

2) Analysis

3) Design

→ for Linear systems :-

$x \rightarrow$ state vector

$y \rightarrow$ output

$u \rightarrow$ Control input

$$\dot{x} = Ax + Bu \Rightarrow f(x, u)$$

$$Y = Cx + \underbrace{Du}_{\rightarrow 0}$$

$A, B, C, D \rightarrow$ system matrices

⇒ In discrete form

$$x((K+1)T) = A_d\, x(KT) + B_d\, U(KT)$$

$$y(KT) = C_d\, x(KT) + D_d\, V(KT)$$

$$A_d \neq A \; ; \; C_d \neq C ; \; D_d \neq D \; ; \; B_d \neq B$$

* Representation of state variable Model:-

1) Controller Canonical form.

2) observer Canonical form.

3) Parallel / Diagonal form.

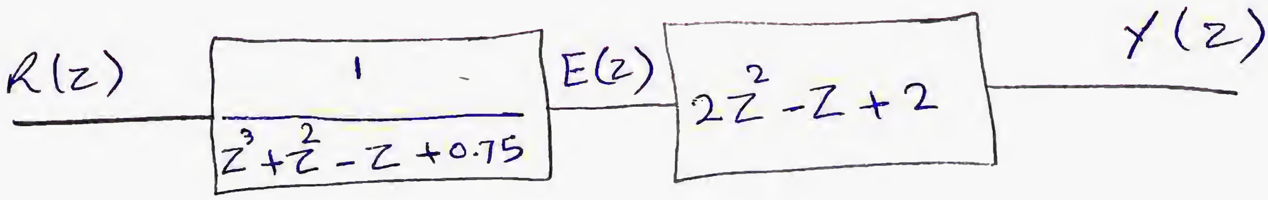
4) Cascaded form.

Ex Given $\dfrac{Y(z)}{R(z)} = \dfrac{2z^2 - z + 2}{z^3 + z^2 - z + 0.75}$

Required 1) Controller Canonical

2) observer Canonical

1) Controller



1 st Block

$$\frac{E(z)}{R(z)} = \frac{1}{z^3 + z^2 - z + 0.75}$$

$$z^3 E(z) + z^2 E(z) - z E(z) + 0.75 E(z) = R(z)$$

Inverse of Z.T

$$e(k+3) + e(k+2) - e(k+1) + 0.75\,e(k) = r(k)$$

Let: $e(k) = x_1 \longrightarrow e(k+1) = x_1(k+1)$

$e(k+1) = x_2 \longrightarrow e(k+2) = x_2(k+1)$

$e(k+2) = x_3 \longrightarrow e(k+3) = x_3(k+1)$

$$e(k+3) = r(k) - e(k+2) + e(k+1) - 0.75\,e(k)$$

$$x_1(k+1) = x_2(k)$$
$$x_2(k+1) = x_3(k)$$
$$x_3(k+1) = r(k) - x_3(k) + x_2(k) - 0.75\,x_1(k)$$

$$x(k+1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.75 & 1 & -1 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(k)$$

لم عكس إشارات المعامل

لاحظ الجزء الى عليه مربع

$$\frac{Y(z)}{E(z)} = 2z^2 - z + 2$$

$$Y(z) = 2z^2 E(z) - z E(z) + 2E(z)$$

$\downarrow$ $z^{-1}.T$

$$y(K) = 2e(K+2) - e(K+1) + 2e(K)$$

Assumption

$$\left.\begin{aligned} X_1(K) &= e(K) \\ X_2(K) &= e(K+1) \\ X_3(K) &= e(K+2) \end{aligned}\right\} \quad y = 2X_3(K) - X_2(K) + 2X_1(K)$$

$$y(K) = \begin{bmatrix} 2 & -1 & 2 \end{bmatrix} X(K)$$

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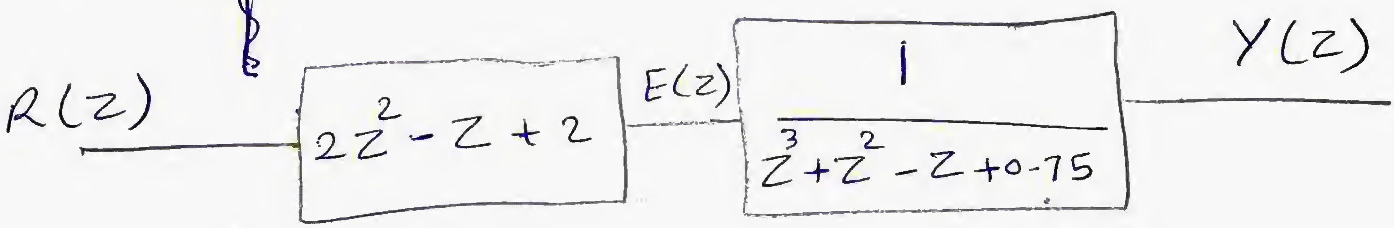
b) observer Canonical form



1st block

$$\frac{E(z)}{R(z)} = 2z^2 - z + 2$$

$$E(z) = 2z^2 R(z) - z R(z) + 2R(z) \quad \downarrow z^{-1} \cdot T$$

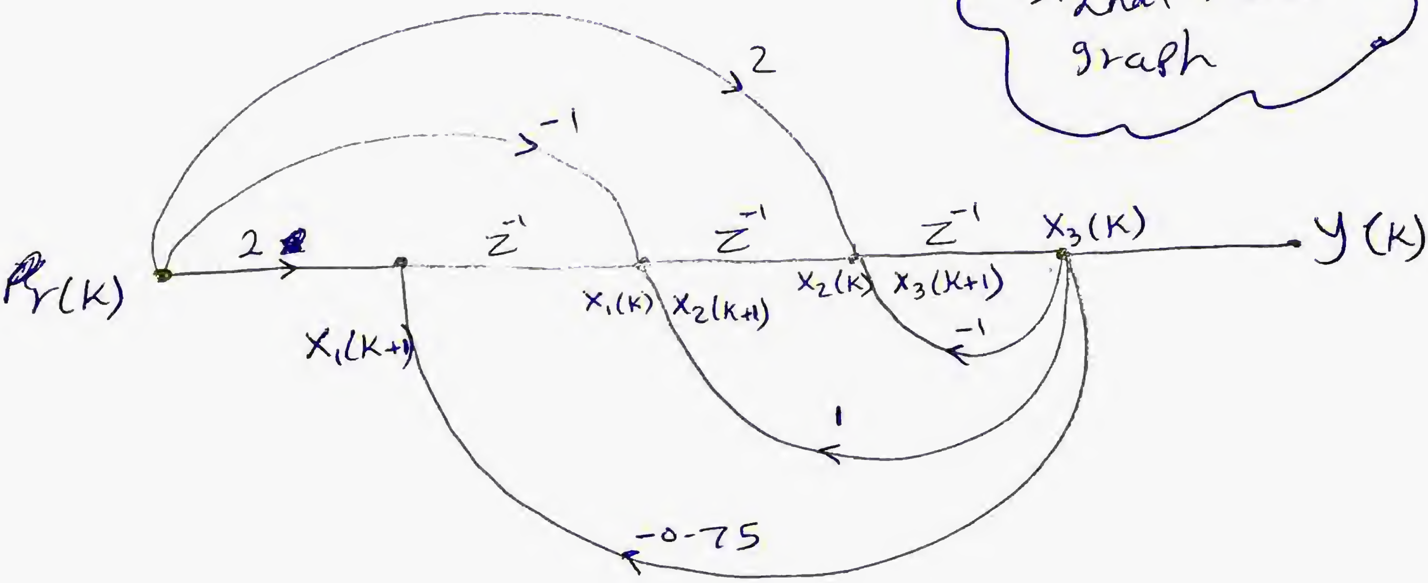
$$\boxed{e(K) = 2r(K+2) - r(K+1) + 2r(K)} \rightarrow (1)$$

$$\frac{Y(z)}{R(z)} = \frac{2z^2 - z + 2}{z^3 + z^2 - z + 0.75} \times \frac{z^{-3}}{z^{-3}}$$

$$= \frac{2z^{-1} - z^{-2} + 2z^{-3}}{1 + z^{-1} - z^{-2} + 0.75 z^{-3}}$$

$$= \frac{2z^{-1} - z^{-2} + 2z^{-3}}{1 - \left[-z^{-1} + z^{-2} - 0.75 z^{-3}\right]}$$

signal flow graph

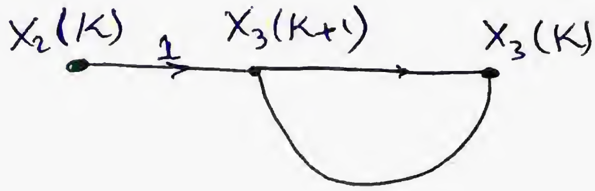


$$x_1(K+1) = -0.75\, X_3(K) + 2\, r(K)$$

$$X_2(K+1) = X_1(K) - r(K) + X_3(K)$$

$$X_3(K+1) = X_2(K) + 2r(K) - X_3(K)$$

← في الرسم السابق مثلاً نقطة $X_3(K+1)$ ، $X_2(K)$ بينهم gain =1 لكنهم مش نقطة واحدة



$$X(K+1) = \begin{bmatrix} 0 & 0 & -0.75 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} X(K) + \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} r(K)$$

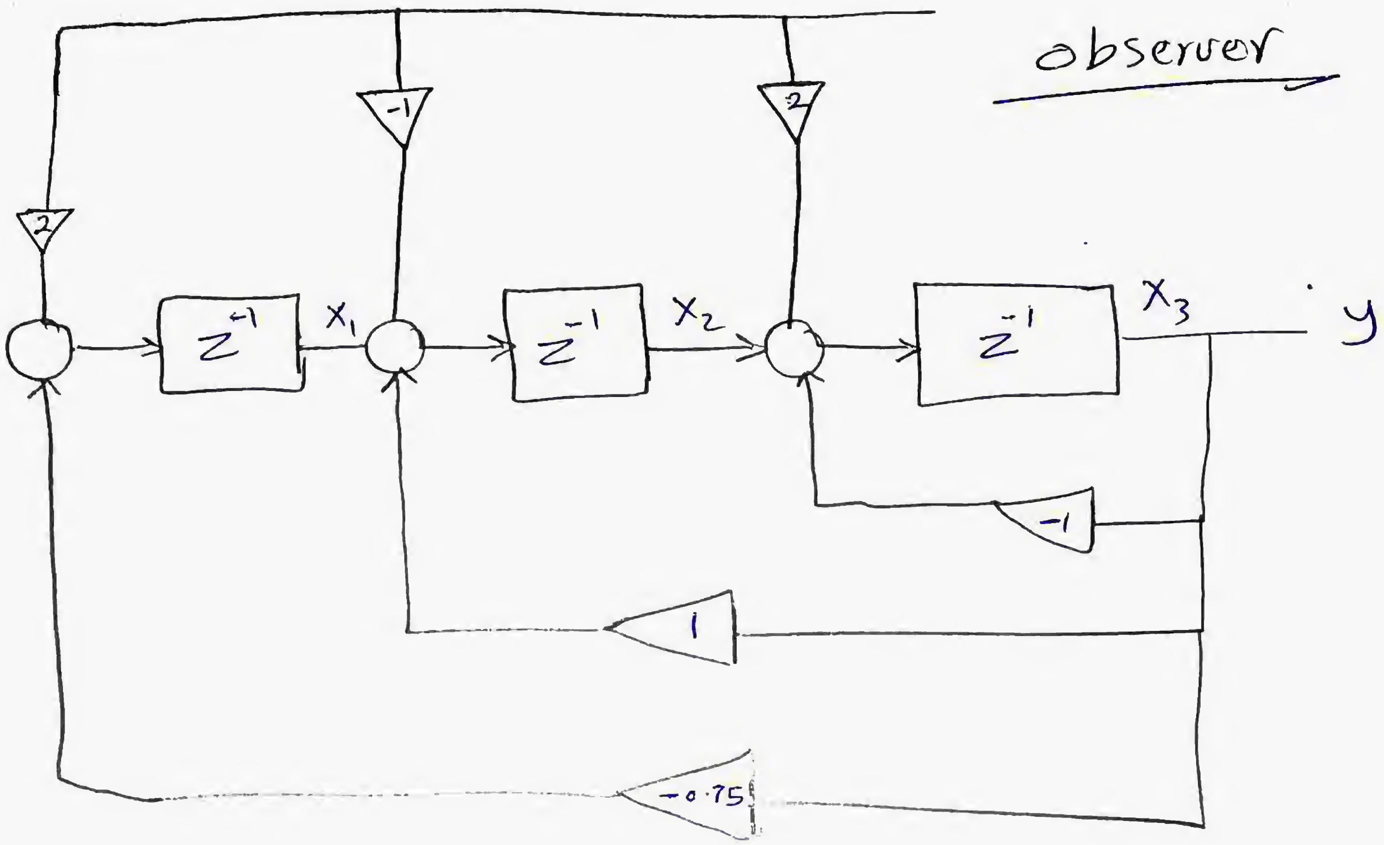
معاملات النظام

$$y(K) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} X(K)$$

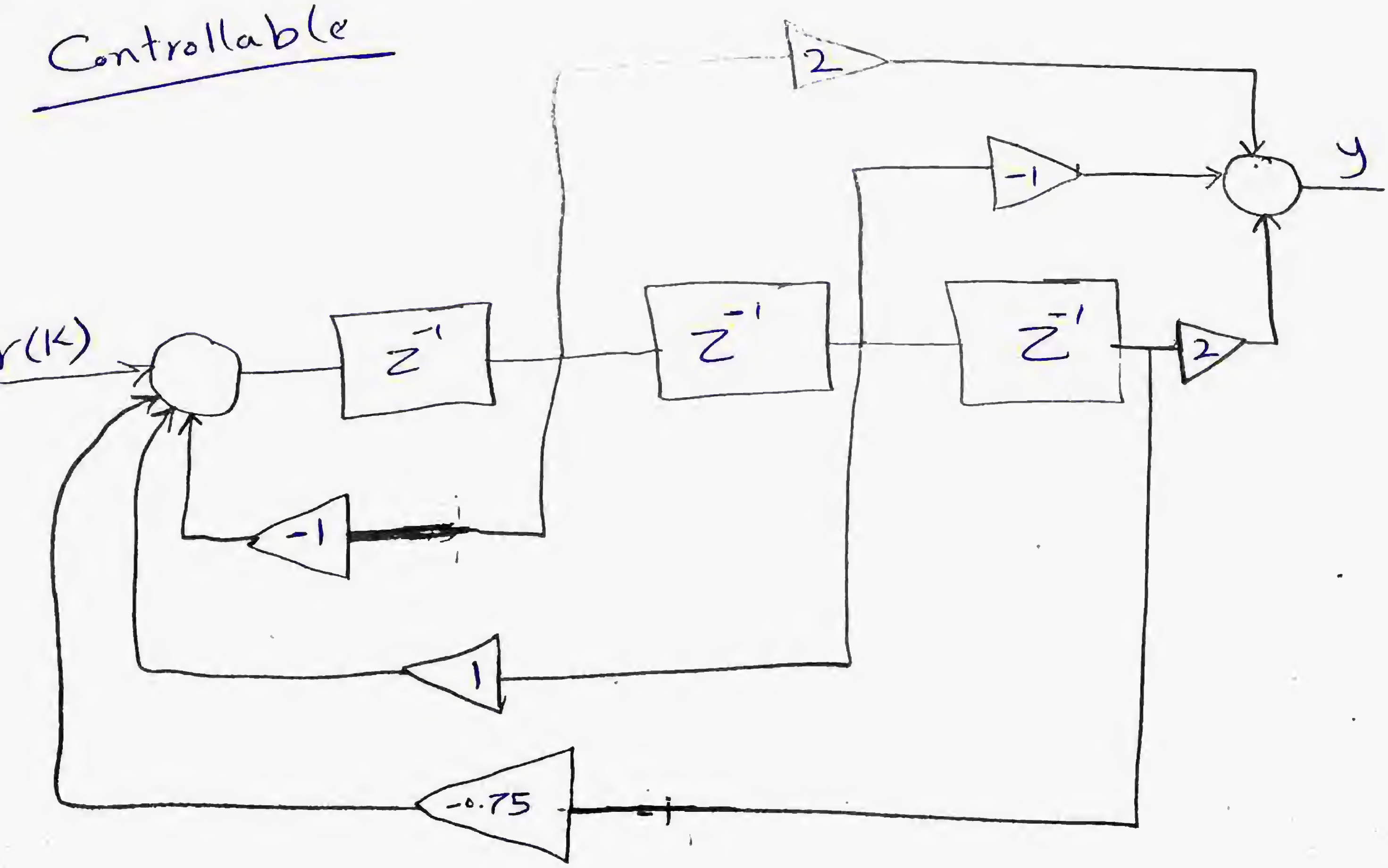
$$\frac{Y(z)}{R(z)} = \frac{\sum P_i}{1 - [\text{loops}]}$$

$$= \frac{2z^{-1} - z^{-2} + 2z^{-3}}{1 - [-z^{-1} + z^{-2} - 0.75 z^{-3}]}$$

observer



Controllable



Given T.F

$$\frac{Y(z)}{R(z)} = \frac{z+0.5}{(z-1)(z-0.1)(z+0.4)}$$

Representation in Diagonal form:-

$$\frac{Y(z)}{R(z)} = \frac{A}{z-1} + \frac{B}{z-0.1} + \frac{C}{z+0.4}$$

$$= \frac{1.19}{z-1} - \frac{1.333}{z-0.1} + \frac{0.143}{z+0.4}$$

$$Y(z) = \underbrace{\frac{1.19}{z-1}R(z)}_{Y_1(z)} + \underbrace{\frac{-1.333}{z-0.1}R(z)}_{Y_2(z)} + \underbrace{\frac{0.143}{z+0.4}R(z)}_{Y_3(z)}$$

$$Y_1(z) = \frac{1.19}{z-1}R(z) \Rightarrow zX_1(z) - Y(z) = 1.19\,R(z)$$

$$y_1(K+1) - y_1(K) = 1.19\,r(K)$$

Let $y_1(K) = x_1(K) \Rightarrow x_1(K+1) = y_1(K+1)$

$$\boxed{x_1(K+1) = x_1(K) + 1.19\,r(K)} \rightarrow (1)$$

$$Y_2(z) = \frac{-1.333}{z-0.1} R(z)$$

$$z Y_2(z) - 0.1\, Y(z) = -1.33\, R(z)$$

$$y_2(k+1) - 0.1\, y_2(k) = -1.33\, r(k)$$

Let $X_2(k) = y_2(k) \Rightarrow X_2(k+1) = y_2(k+1)$

$$\boxed{X_2(k+1) = 0.1\, X_2(k) - 1.33\, r(k)} \longrightarrow (2)$$

$$Y_3(z) = \frac{0.143}{z+0.4} R(z)$$

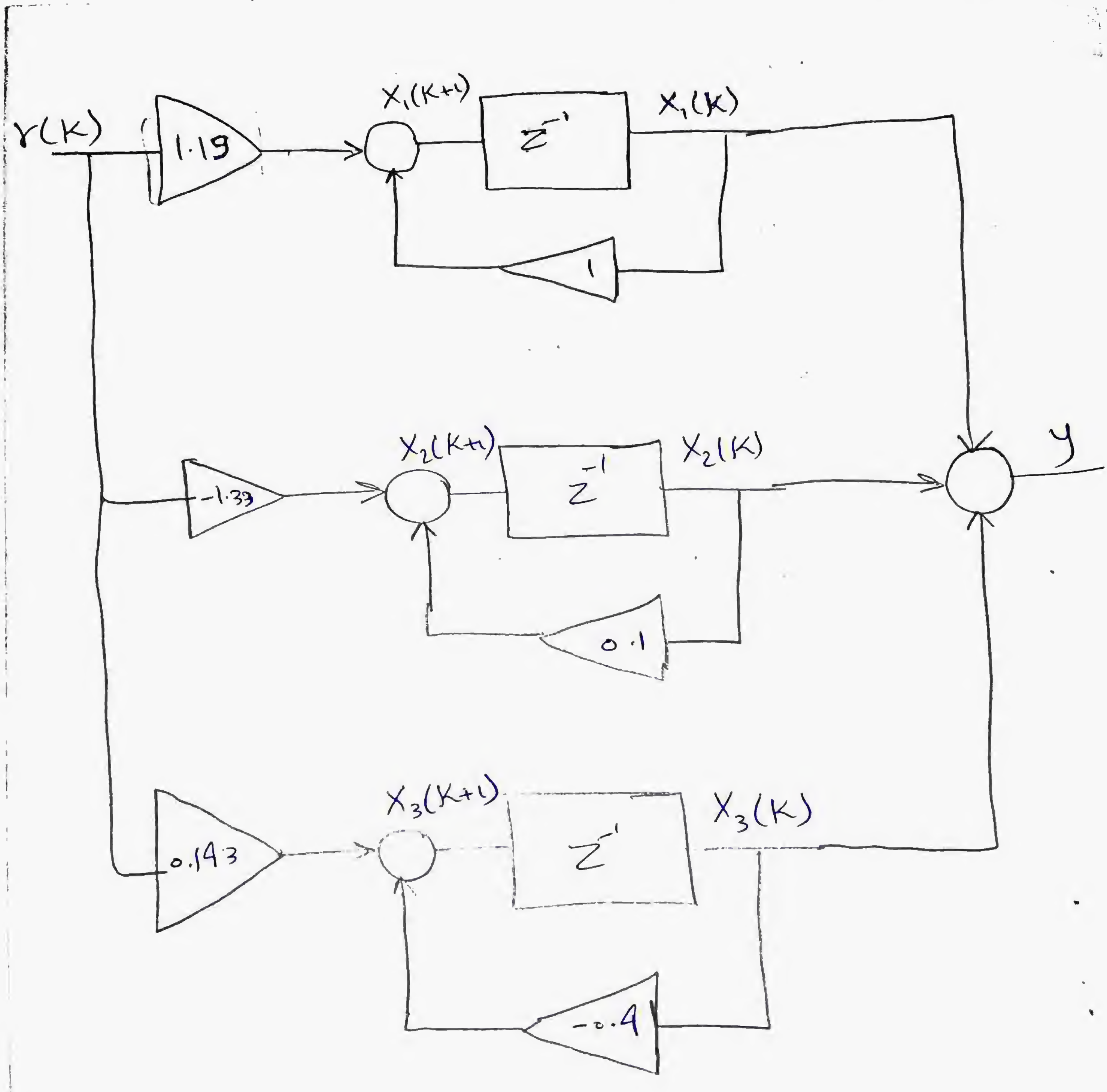
$$z Y_3(z) + 0.4\, Y_3(z) = 0.143\, R(z)$$

$\vdots$

$$\boxed{X_3(k+1) = -0.4\, X_3(k) + 0.143\, r(k)} \longrightarrow (3)$$

$$X(k+1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & -0.4 \end{bmatrix} + \begin{bmatrix} 1.19 \\ -1.33 \\ 0.143 \end{bmatrix} r(k)$$

$$y(k) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} X(k)$$


r(K)
1.19
X1(K+1)
Z^-1
X1(K)
1
-1.33
X2(K+1)
Z^-1
X2(K)
0.1
y
0.143
X3(K+1)
Z^-1
X3(K)
-0.4